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Formation and Evolution of Black Holes in Galactic Nuclei and Star Clusters

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This project studies the formation, growth, and co-evolution of single and multiple supermassive black holes (SMBHs) and compact objects like neutron stars, white dwarfs, and stellar mass black holes in galactic nuclei and star clusters, focusing on the role of stellar dynamics. In this paper we focus on one exemplary topic out of a wider range of work done, the study of orbital parameters of binary black holes in galactic nuclei (binding energy, eccentricity, relativistic coalescence) as a function of initial parameters. In some cases the classical evolution of black hole binaries in dense stellar systems drives them to surprisingly high eccentricities, which is very exciting for the emission of gravitational waves and relativistic orbit shrinkage. Such results are interesting to the emerging field of gravitational wave astronomy, in relation to a number of ground and space based instruments designed to measure gravitational waves from astrophysical sources (VIRGO, Geo600, LIGO, LISA). Our models self-consistently cover the entire range from Newtonian dynamics to the relativistic coalescence of SMBH binaries.

1 Introduction

MBH formation and their interactions with their host galactic nuclei is an important ingredient for our understanding of galaxy formation and evolution in a cosmological context, e.g. for predictions of cosmic star formation histories or of MBH demographics (to predict events which emit gravitational waves). If galaxies merge in the course of their evolution, there should be either many binary or even multiple black holes, or we have to find out what happens to black hole multiples in galactic nuclei, e.g. whether they come close enough together to merge under emission of gravitational waves, or whether they eject each other in gravitational slingshot. For numerical simulations of the problem all models depend on an unknown scaling behaviour, because the simulated particle number is not yet realistic due to limited computing power[25,26,22,6]. Dynamical modelling of non-spherical dense stellar systems (with and without central BH) is even less developed than in the spherical case. Here we present a set of numerical models of the formation and evolution of binary black holes in rotating galactic nuclei. Since we are interested in the dynamical evolution of MBH binaries in their final phases of evolution (the last parsec problem) we somehow abstract from the foregoing complex dynamics of galactic mergers. We assume that after some violent dynamic relaxation a typical initial situation consists of a spherical or axisymmetric coherent stellar system (galactic nucleus), where fluctuations in density and potential due to the galaxy merger have decayed, which is reasonable on an (astrophysically) short time scale of a few ten million years. The MBHs, which were situated in the centre of each of the previously merged galaxies, are located at the boundary of the dense stellar core, some few hundred parsec apart. This situation is well observable[14].

According to the standard theory, the subsequent evolution of the black holes is divided in three intergradient stages[5]: 1. Dynamical friction causes a transfer of the black holes' kinetic energy to the surrounding field stars, the black holes spiral to the centre where they form a binary. 2. While hardening, the effect of dynamical friction reduces and the evolution is dominated by superelastic scattering processes, that is the interaction with field stars closely encountering or intersecting the binaries' orbit, thereby increasing the binding energy. 3. Finally the black holes coalesce throughout the emission of gravitational radiation, potentially detectable by the planned space-based gravitational wave antennae LISA.

In this paper, the behaviour of the orbital elements of a black hole binary in a dense stellar system is investigated in a self-consistent way from the beginning till the relativistic merger and its emission of gravitational waves. The evolution of the eccentricity has been discussed for some time[21,12,25,6,22]. According to Peters & Mathews and Peters[28,27] the timescale of coalescence due to the emission of gravitational radiation is given by

$$t_{gr} = \frac{5}{64} \frac{c^5 a_{gr}^4}{G^3 M_1 M_2 (M_1 + M_2) F(e)} \tag{1}$$

wherein M_1, M_2 denote the black hole masses, a_{gr} the characteristic separation for gravitational wave emission, G the gravitational constant, c the speed of light and

$$F(e) = (1 - e^2)^{-7/2} \left(1 + \frac{73}{24} e^2 + \frac{37}{96} e^4\right) \tag{2}$$

a function with strong dependence on the eccentricity e. Thus the coalescence time can shrink by several orders of magnitude if the eccentricity is high enough, resulting in a strengthened burst of gravitational radiation. Highly eccentric black hole binaries would represent appropriate candidates for forthcoming verification of gravitational radiation through the planned mission of the Laser Interferometer Space Antenna mission LISA.

2 Numerical Method, Initial Models

The simulations have been performed using NBODY6++, a parallelized version of Aarseth's NBODY6[1,35,2]. The code includes a Hermite integration scheme, KS-regularization[15] and the Ahmad-Cohen neighbour scheme[4]. No softening of the interaction potential of any two bodies is introduced; this allows an accurate treatment of the effects due to superelastic scattering events, which play a crucial part in black hole binary evolution and require a precise calculation of the trajectories throughout the interaction. The code and its parallel performance has been described in detail in this series and elsewhere[35,13]. The survey has been carried out for a total particle number of up to $N = 1\ 000\ 000$ including two massive black holes with $M_1 = M_2 = 0.01$ embedded in a dense stellar system of equal-mass particles $m_* \approx 1.0 \cdot 10^{-6}$. The total mass of the system is normalized to unity. The initial stellar distribution was taken from generalized King models with and without rotation[16,19,11].

3 Simulations

3.1 Newtonian Evolution of the Binary Black Hole

In the first evolutionary stage, each black hole individually suffers dynamical friction with the surrounding low mass stars, which is the main process of losing energy. The role of dynamical friction decreases when a permanently bound state occurs, as the dynamical friction force acts preliminary on the motion of the now formed binary rather than on the individual black holes. Superelastic scattering events of field stars at the binary then become more and more important for the reduction of its energy. The process sustains an ongoing “hardening” of the binary (shrinking of semi-major axis and increase of energy) and also in many cases a high eccentricity. While the hardening rates are well understood[33,30] and do not depend much on the initial parameters of the preceding galactic merger, this is not as clear for the eccentricity, which depends on initial conditions at least to some extent[10].

In a spherically symmetric system the binary hardening would stall after a few crossing times, because loss-cone orbits of stars, which come close to the cenral SMBH binary will be depleted, and replenishment takes place only on a much longer relaxation time. This effect is more dramatic for systems with large particle number, because the relaxation time increases strongly, and is depicted on the top panel of Fig. 1; it has been claimed that in real galaxies with their very large particle numbers therefore the SMBH binary will not reach relativistic coalescence. This situation was relaxed from two sides, first by a careful analysis of loss-cone refilling time scales combining direct N-body and Fokker-Planck models[23], and by looking for a moderately rotating, axisymmetric galactic nucleus[7], where the loss cone remains full even for large particle numbers (see lower panel in Fig. 1). Since some degree of perturbation of a spherical model is quite natural for a remnant after galactic mergers, many of them might even be triaxial rather than axisymmetric, the stalling problem does not exist anymore.

4 Relativistic Dynamics of Black Holes in Galactic Nuclei

4.1 Introduction

Relativistic stellar dynamics is of paramount importance for the study of a number of subjects. For instance if we want to have a better understanding of what the constraints on alternatives to supermassive black holes are; in order to canvass the possibility of ruling out stellar clusters, one must do detailed analysis of the dynamics of relativistic clusters. Furthermore the dynamics of compact objects around SMBH and of multiple SMBH in

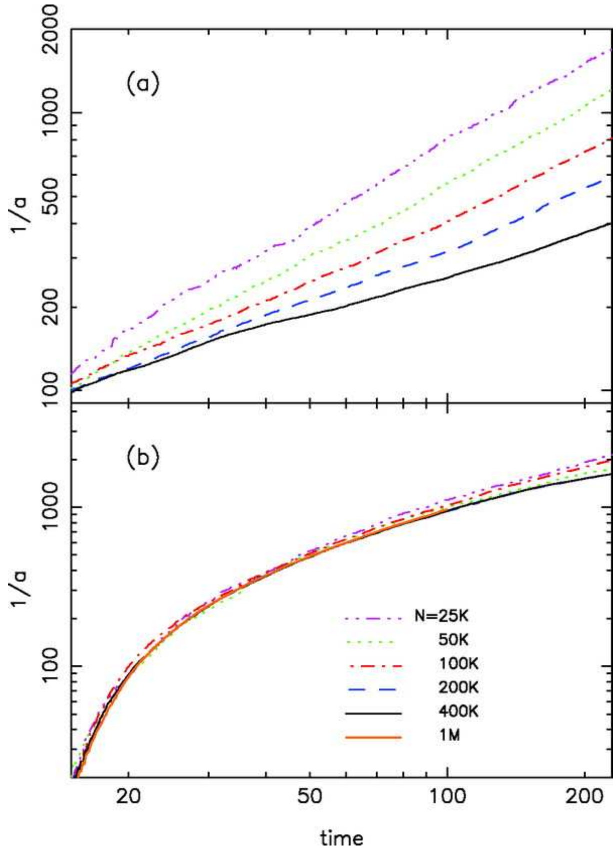

Figure 1. Evolution of the inverse semi-major axis of a black hole binary in direct N-body models with varying particle number N - top panel: for spherically symmetric systems a stalling occurs; - bottom panel: for axisymmetric rotating systems there is no sign of stalling. Compare[7].

galactic nuclei requires the inclusion of relativistic effects. Our current work deals with the evolution of two SMBHs, bound to each other, and looking at the phase when they get close enough to each other that relativistic corrections to Newtonian dynamics become important, which ultimately lead to gravitational radiation losses and coalescence.

Efforts to understand the dynamical evolution of a stellar cluster in which relativistic effects may be important have been already done by[17],[32],[31] and[18]. In the earlier work $1\mathcal{PN}$ and $2\mathcal{PN}$ terms were neglected[18] and the orbit-averaged formalism[27] used. We describe here a method to deal with deviations from Newtonian dynamics more rigorously than in most existing literature (but compare[24,3], which are on the same level of PNaccuracy). We modified the NBODY6++ code to allow for post-Newtonian ($\mathcal{PN}$) effects of two particles getting very close to each other, implementing in it the $1\mathcal{PN}$, $2\mathcal{PN}$ and $2.5\mathcal{PN}$ corrections fully from[34,37].

4.2 Method: Direct Summation NBODY with Post-Newtonian Corrections

The version of direct summation NBODY method we employed for the calculations, NBODY6++, includes the *KS regularisation*. This means that when two particles are tightly bound to each other or the separation among them becomes smaller during a hyperbolic encounter, the couple becomes a candidate for a regularisation in order to avoid problematical small individual time steps[15]. We modified this scheme to allow for relativistic corrections to the Newtonian forces by expanding the acceleration in a series of powers of $1/c$ in the following way[9,34]:

$$\underline{a} = \underbrace{\underline{a}_0}_{\text{Newt.}} + \underbrace{\underbrace{c^{-2}\underline{a}_2}_{1\mathcal{PN}} + \underbrace{c^{-4}\underline{a}_4}_{2\mathcal{PN}}}_{\text{periastron shift}} + \underbrace{\underbrace{c^{-5}\underline{a}_5}_{2.5\mathcal{PN}}}_{\text{grav. rad.}} + \mathcal{O}(c^{-6}), \tag{3}$$

where $\underline{a}$ is the acceleration of particle 1, $\underline{a}_0 = -Gm_2\underline{n}/r^2$ is the Newtonian acceleration, G is the gravitation constant, m_1 and m_2 are the masses of the two particles, r is the distance of the particles, $\underline{n}$ is the unit vector pointing from particle 2 to particle 1, and the $1\mathcal{PN},\ 2\mathcal{PN}$ and $2.5\mathcal{PN}$ are post-Newtonian corrections to the Newtonian acceleration, responsible for the pericenter shift ($1\mathcal{PN},\ 2\mathcal{PN}$) and the quadrupole gravitational radiation ($2.5\mathcal{PN}$), correspondingly, as shown in Eq. (3). As an example we give the expressions for the $1\mathcal{PN}$ and $2.5\mathcal{PN}$ terms, for $2\mathcal{PN}$ see the cited literature[34]:

$$\begin{aligned}\underline{a}_2 = \frac{Gm_2}{r^2}\Bigg\{\underline{n}\Big[& -v_1^2 - 2v_2^2 + 4v_1v_2 + \tfrac{3}{2}(nv_2)^2 \\ & +5\big(\tfrac{Gm_1}{r}\big) + 4\big(\tfrac{Gm_2}{r}\big)\Big] + (\underline{v}_1 - \underline{v}_2)\big[4nv_1 - 3nv_2\big]\Bigg\}\end{aligned} \tag{4}$$

$$\begin{aligned}\underline{a}_5 = \tfrac{4}{5}\tfrac{G^2m_1m_2}{r^3}\Bigg\{ & (\underline{v}_1 - \underline{v}_2)\Big[-(\underline{v}_1 - \underline{v}_2)^2 + 2\big(\tfrac{Gm_1}{r}\big) - 8\big(\tfrac{Gm_2}{r}\big)\Big] \\ & +\underline{n}(nv_1 - nv_2)\big[3(\underline{v}_1 - \underline{v}_2)^2 - 6\big(\tfrac{Gm_1}{r}\big) + \tfrac{52}{3}\big(\tfrac{Gm_2}{r}\big)\big]\Bigg\}.\end{aligned} \tag{5}$$

In the last expressions $\underline{v}_1$ and $\underline{v}_2$ are the velocities of the particles. For simplification, we have denoted the vector product of two vectors, $\underline{x}_1$ and $\underline{x}_2$, like x_1x_2. We integrated our correcting terms as external forces into the *two-body KS regularisation* scheme which requires to compute their time derivatives in the Hermite scheme as well (not shown in equations here for brevity).

4.3 First Results

In Fig. 2 the impact of relativistic, Post-Newtonian dynamics to the separation of the binary black holes in our simulations is seen. The curve deviates from the Newtonian results when gravitational radiation losses set in and cause a sudden coalescence ($1/a \to \infty$) at a finite time. Gravitational radiation losses are supported by the high eccentricity of the SMBH binary. It is interesting to note that the inclusion or exclusion of the conservative 1 and 2PN terms changes the coalescence time considerably. Details of these results will be published elsewhere[8].

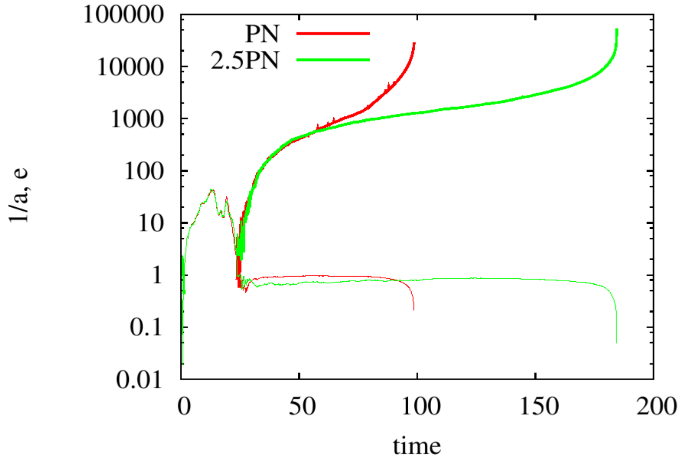

Figure 2. Effect of Post-Newtonian (PN) relativistic corrections on the dynamics of black hole binaries in galactic nuclei, plotted are inverse semi-major axis and eccentricity as a function of time. The red line uses the full set of PN corrections, while the green line has been obtained by artificially only using the dissipative $\mathcal{PN}$ 2.5 terms. Here $c = 457$ has been chosen in model units, see more details in[8].

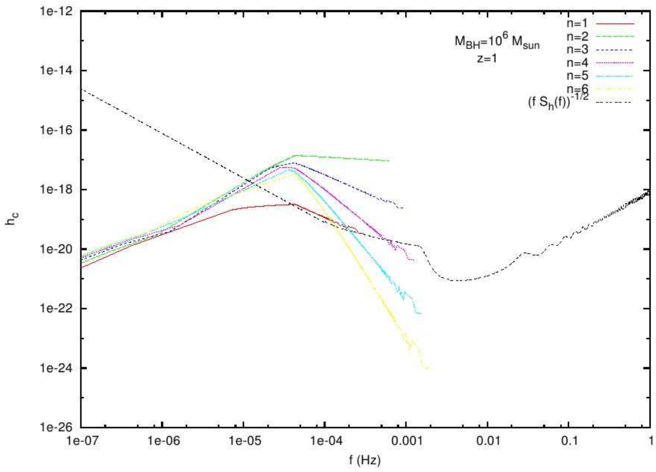

Figure 3. Locus of SMBH binaries from our simulations in the LISA sensitivity diagram during their final inspiral and coalescence. Plotted is dimensionless strain versus frequency, for all relevant harmonics (circular orbit: $n = 2$ dominates), and the LISA sensitivity curve. We have selected as an example here the signal expected from a SMBH binary of one million solar masses located at a redshift of $z \approx 1$. Further details will be published elsewhere[29].

Once the SMBH binary starts to dramatically lose binding energy due to gravitational radiation its orbital period will drop from a few thousand years to less than a year very quickly (timescale much shorter than the dynamical time scale in the galactic center, which defines our time units). Then the SMBH binary will enter the LISA band, i.e. its gravitational radiation will be detectable by LISA. LISA, Laser Interferometer Space Antenna, is a system of three space probes with laser interferometers to measure gravitational waves, see e.g. `http://lisa.esa.int/`. Once our SMBH binary decouples from the rest of the system we just follow its relativistic two-body evolution, starting with exactly the orbital parameters (eccentricity!) as they came out from the N-body model. It is then possible to predict the gravitational radiation of our SMBH binary relative to the LISA sensitivity curve, which is depicted in Fig. 3. Plotted are different harmonics of the gravitational radiation, for the circular orbit $n = 2$ is dominant, while for eccentric orbits higher harmonics are stronger[28,27]. One can see in the plot how the SMBH binary enters into the LISA sensitivity regime with some eccentricity.

5 Summary

We have shown that supermassive black hole binaries in galactic nuclei may reach the stalling barrier and will reach the relativistic coalescence phase in a timescale shorter than the age of the universe. A gravitational wave signal expected for the LISA satellite from these SMBH binaries is expected, in particular due to the high eccentricity of the SMBH binary when entering the relativistic coalescence phase. Our models cover self-consistently the transition from the Newtonian dynamics to the situation when relativistic, Post-Newtonian ($\mathcal{PN}$) corrections start to influence the relative SMBH motion. After the shrinking time scale became very short the binary decouples from the rest of the galactic nucleus and can be treated as a relativistic two-body problem. We follow this evolution formally to the coalescence of the two black holes using $\mathcal{PN}$ terms of up to order $2.5\mathcal{PN}$ and determine the gravitational wave emission in different modes relative to the LISA sensitivity curve.

This paper has only selected one of the highlights of a number of applications of our parallel direct N-body code NBODY6++, because of constraints of space. Other projects followed here are the detailed modelling of populations and spectra, as well as gravitational wave emission by neutron stars, black holes and white dwarfs from globular clusters (for ground based detectors such as VIRGO, LIGO, GEO600) (by A. Borch, J. Downing), the study of inspiralling globular clusters onto the SMBH in our Galaxy (by A. Ernst, A. Just[36]), star-disk interactions in thick accretion disks around SMBH (by C. Omarov) and modelling of loss cones and tidal disruption near a single black hole (by O. Porth). Last but not least direct N-body models of galactic nuclei need to be complemented still by statistical models to reach realistically high particle numbers - here we use an orbit averaged 2D Fokker-Planck equation (by J. Fiestas).

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